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NEW STUDY ON THE CONVERGENCE OF A FORMAL TRANSFORMATION, II

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Abstract. A system of two nonlinear differential equations with an irregular type singularity not satisfying the Poincaré condition is studied. A two-parameter family of bounded solutions is constructed by the fixed point technique. The domain of holomorphy of the set of functions appearing in the fixed point technique is to be given by a family of the product of two circles over every point in a domain of independent variable. The radius of one circle depends on the argument of the independent variable only, while that of the other essentially depends on the independent variable itself.

1. Introduction.

1°. Assumptions. In a previous paper [6], the author studies a system of two nonlinear differential equations of the form

$$x^2\frac{dy}{dx}=(\mu+\alpha x)y+f(x, y, z)\,,\qquad x^2\frac{dz}{dx}=(-\nu+\beta x)z+g(x, y, z)\,, \tag{A}$$

under the assumptions that

(i) x is an independent variable;

(ii) μ and ν are positive numbers and their ratio is irrational;

(iii) α and β are complex constants and there is a positive quantity κ satisfying the inequalities

$$\mu+\kappa\Re\alpha>0\,,\qquad -\nu+\kappa\Re\beta>0\,; \tag{1.1}$$

(iv) $f(x, y, z)$ and $g(x, y, z)$ are holomorphic and bounded functions of (x, y, z) for

$$|x|<a\,,\qquad |y|<b\,,\qquad |z|<b\,, \tag{1.2}$$

and their Taylor series expansions in (y, z) contain neither the constant terms nor the linear terms, where a and b are small positive constants.

2°. Review of a previous result. Under these assumptions, the following was proved:

PROPOSITION 1. *Let ε_0 be a preassinged sufficiently small positive number. There exists a formal transformation of the form*

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