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POLARIZED SURFACES OF LOW DEGREES WITH RESPECT TO THE DELTA-GENUS

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Abstract. We classify such polarized surfaces that a certain equality holds between the self-intersection number and the delta-genus and that the complete linear system has finite base locus and defines a non-birational map. The surface obtained by the blowing up at a point of such a surface turns out to be a double cover of a desingularization of a surface with delta-genus zero. We classify these surfaces according to the shape of the inverse image of the image of the exceptional curve. Six of the classes consist of fiber spaces over the projective line and the other class consists of irrational ruled surfaces. Conversely, we show the existence of polarized surfaces in each of the seven classes.

1. Introduction. Let (M, L) be a polarized manifold, i.e., a pair of an n-dimensional complete algebraic manifold M and an ample Cartier divisor L on it. First we recall some definitions and necessary results. The integers $\chi_j(M, L)$ $(j=0, \dots, n)$ are the coefficients of the Hilbert polynomial

$$\chi(M, tL)=\sum\chi_j(M, L)\frac{t^{[j]}}{j!},$$

where $t^{[0]}=1$ and $t^{[j]}=t(t+1)\cdots(t+j-1)$ $(j>0)$. The *sectional genus* of (M, L) is defined as

$$g(M, L):=1-\chi_{n-1}(M, L).$$

By the Riemann-Roch theorem we get $2g(M, L)-2=L^{n-1}\cdot((n-1)L+K_M)$, where K_M is a canonical divisor of M. We define the *Δ-genus* as

$$\Delta(M, L):=n+L^n-h^0(M, L).$$

A prime divisor R_{n-1} in the linear system $|L|$ is called a *rung* of (M, L). We have $g(M, L)=g(R_{n-1}, L_{|R_{n-1}})$. If the restriction map

$$r_{n-1}: H^0(M, \mathcal{O}(L))\to H^0(R_{n-1}, \mathcal{O}(L_{|R_{n-1}}))$$

is surjective, then R_{n-1} is said to be *regular*. A rung R_{n-1} is regular if and only if $\Delta(M, L)=\Delta(R_{n-1}, L_{|R_{n-1}})$. We denote by $\mathrm{Bs}|L|$ the base locus of the linear system $|L|$.

As to the existence of a regular rung, Fujita proved the following:

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