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PERIODIC SOLUTIONS OF THE NAVIER-STOKES EQUATIONS IN UNBOUNDED DOMAINS

Dedicated to Professor Kôji Kubota on his sixtieth birthday

HIDEO KOZONO AND MITSUHIRO NAKAO

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Abstract. We shall construct a periodic strong solution of the Navier-Stokes equations for the prescribed external force in unbounded domains.

Introduction. The purpose of this paper is to show that if the incompressible fluid in *unbounded* domains is governed by the periodic external force, the Navier-Stokes equations have a *periodic strong* solution with the same period as the external force. Let Ω be a domain in $\boldsymbol{R}^n$ $(n \geq 3)$, not necessarily bounded, with smooth boundary $\partial\Omega$. Consider the following Navier-Stokes equations in Ω:

$$\text{(N-S)} \qquad \begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + u \cdot \nabla u + \nabla p = f, & x \in \Omega, \quad t \in \boldsymbol{R}, \\ \operatorname{div} u = 0, & x \in \Omega, \quad t \in \boldsymbol{R}, \\ u|_{\partial\Omega} = 0, \end{cases}$$

where $u = u(x, t) = (u^1(x, t), \ldots, u^n(x, t))$ and $p = p(x, t)$ denote the unknown velocity vector and pressure of the fluid at point $(x, t) \in \Omega \times \boldsymbol{R}$, respectively; while $f = f(x, t) = (f^1(x, t), \ldots, f^n(x, t))$ is the given periodic external force.

Under some restrictive conditions, Serrin [20] gave a criterion for the existence of periodic solutions of (N-S) when Ω is a three-dimensional bounded domain whose boundary moves periodically in time. Kaniel-Shinbrot [11] considered a simpler case such as bounded domains whose boundary is fixed in time and realized the criterion of Serrin. Having introduced the notion of reproductive property, they showed the existence of periodic strong solutions with periodic small forces f. In two-dimensional bounded domains, Takeshita [23] obtained the same result as Kaniel-Shinbrot [11] without assuming the smallness of f. The original problem posed by Serrin had been treated by Morimoto [19] and Miyakawa-Teramoto [18] who showed the existence of periodic weak solutions. Later on, Teramoto [25] constructed periodic strong solutions in a situation such that the boundary moves slowly in time.

All of these results are obtained in two- or three-dimensional *bounded* domains. On the other hand, few results are known in *unbounded* domains. Recently, Maremonti

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