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NONLINEAR OSCILLATIONS IN A DISCRETE DIFFUSIVE NEUTRAL LOGISTIC EQUATION

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Abstract. We consider the dynamics of a logistic neutral delay system which is continuous in time and discrete in space. Such a system models the growth of a single-species population distributed over a ring of identical patches and it allows for population dispersing from one patch to its nearest neighbors. We shall show that (i) in the case of instantaneous dispersion feedback, the dispersal in the local growth rate and the neutral term have a stablizing effect on the population dynamics; (ii) increasing the delay in the growth phase changes the stability of a positive equilibrium and leads to a Hopf bifurcation of synchronous or phase-locked oscillations if the dispersion is small; (iii) the neutral term may bring about several global branches of phase-locked oscillations which would not occur in the absence of a neutral term, and hence the neutral term in this situation has a destablizing influence.

1. Introduction. The purpose of this paper is to consider the dynamics of a logistic neutral delay system which is continuous in time and discrete in space. Such a system models the growth of a single-species population distributed over a ring of identical patches (islands or habitats) and it allows for population dispersing from one patch to its nearest neighbors. We shall study phase-locked oscillations in the model and draw some conclusions about the effect of dispersion as well as the delay and neutral term on population dynamics.

The role of space and dispersal in interactions among biological populations has been the subject of much theoretical and experimental work (cf. [6], [20], [28]–[31], [35], [38] and references therein). It is widely recognized that the spatial heterogeneity of environment, which leads to ecological interactions, operates in general to increase species diversity. For example, it has been asserted that in some cases dispersal can lend stability to interactions (cf. [18], [19], [35], [38], [44]) while in other cases dispersal can also give rise to instability (cf. [28], [35], [38], [44]). For the references related to this subject, we refer to [18], [27], [28], [34], [35] and [43] for the study of Lotka-Volterra models in a spatially heterogeneous environment on persistence and stability, and to [5], [7], [8], [11], [12], [17], [19], [22], [35], [38], [40], [44] and [46] for similar discussions on single-species models in a patchy environment.

Keywords. Neutral logistic equation, delayed growth response, symmetric Hopf bifurcation, stability, single species population, patchy environment, dispersal and diffusion.

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