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EXISTENCE, UNIQUENESS AND ASYMPTOTIC STABILITY OF PERIODIC SOLUTIONS OF PERIODIC FUNCTIONAL DIFFERENTIAL SYSTEMS

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Abstract. We consider here a general Lotka-Volterra type n-dimensional periodic functional differential system. Sufficient conditions for the existence, uniqueness and global asymptotic stability of periodic solutions are established by combining the theory of monotone flow generated by FDEs, Horn's asymptotic fixed point theorem and linearized stability analysis. These conditions improve and generalize the recent ones obtained by Freedman and Wu (1992) for scalar equations. We also present a nontrivial application of our results to a delayed nonautonomous predator-prey system.

1. Introduction. The n-dimensional Lotka-Volterra system takes the following form:

$$\dot{x}_i(t)=x_i(t)\left(b_i+\sum_{j=1}^{n} a_{ij}x_j(t)\right), \tag{1.1}$$

where the dot denotes the differentiation with respect to t and b_i, a_{ij} $(i,j=1,\dots,n)$ are constants. This system has long played an important role in mathematical population biology (cf. Hofbauer and Sigmund [15]). However, realistic models often require the inclusion of effects of time delays and the changing environment. This leads us to the study of the following more general nonautonomous Lotka-Volterra system with time delay:

$$\dot{x}_i(t)=x_i(t)\left(b_i(t)+\sum_{j=1}^{n} a_{ij}(t)x_j(t)+\sum_{j=1}^{n} c_{ij}(t)x_j(t-\tau_{ij}(t))\right), \tag{1.2}$$

where $i=1,\dots,n$. Biologically, due to the difference in species, $\tau_{ij}(t)$ are usually different. However, to minimize the technical complexity in the presentation of our results, we assume in the rest of this paper that $\tau_{ij}(t)=\tau(t)$. Nevertheless, we would like to men-

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