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ON THE IWASAWA INVARIANTS OF CERTAIN REAL ABELIAN FIELDS

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Abstract. For any totally real number field k and any prime number p, the Iwasawa lambda-invariant and the mu-invariant are conjectured to be both zero. We give a new efficient method to verify this conjecture for certain real abelian fields. The new features of our method compared with other existing ones are that we use effectively cyclotomic units and that we introduce a new way to apply p-adic L-functions to the conjecture.

1. Introduction. For a number field k and a prime number p, denote by $\lambda=\lambda_p(k)$ and $\mu=\mu_p(k)$ the Iwasawa λ-invariant and the μ-invariant associated to the ideal class group of the cyclotomic $\boldsymbol{Z}_p$-extension k_∞/k, respectively. For any totally real number field k and any p, it is conjectured that $\lambda_p(k)=\mu_p(k)=0$ (cf. Iwasawa [I3, p. 316], Greenberg [Gr]), which is often called Greenberg's conjecture. We already know that $\mu=0$ when k is abelian over $\boldsymbol{Q}$ (cf. Ferrero-Washington [FW]). When k is a real quadratic field, several authors have given some sufficient conditions for the conjecture to be true mainly in terms of units of the n-th layer k_n of the $\boldsymbol{Z}_p$-extension for some n (cf. [Ca], [Gr], [FK1], [FKW], [F1], [K], [FT], [T] and [FK2]). These conditions are roughly divided into two classes; the case $(\frac{k}{p})=1$ (cf., e.g. [FK1], [FT]), and the other case (cf., e.g. [FK2]). Calculating a system of fundamental units of k_0 or k_1 (cf., e.g. [FK1], [FT]) in the first case, or finding a "good" unit (in the sense of [FK2]) of k_n with $0\leq n\leq 3$ in the second case, they have shown that the conjecture is valid for many real quadratic fields with small discriminants and $p=3$. However, the conjecture is not yet settled, for example, when $k=\boldsymbol{Q}(\sqrt{254})$, $\boldsymbol{Q}(\sqrt{473})$ and $p=3$ (for which $(\frac{k}{p})=-1$). A reason for this is, as Takashi Fukuda kindly informed us, that one is required to have some information on the units of k_n with n at least 5 (!) to apply the criterion of [FK2] to these fields.

The primary purpose of the present paper is to give a simple necessary and sufficient condition (Theorem, Corollary) for the conjecture when k is a real abelian field and $p>2$ for which p does not split in k and the couple (k, p) satisfies some further assumptions (C). It is given in terms of certain cyclotomic units and some polynomials related to a p-adic L-function. From our theorem, it is possible to derive criteria for the conjecture

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