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JULIA SET OF THE FUNCTION $z \exp(z+\mu)$ II

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Abstract. We are concerned with bifurcation of Julia sets for the one-parameter family of functions in the title with the real parameter μ. In particular, the distribution of values of μ, for which the Julia sets of the functions coincide with the complex plane, is discussed.

Introduction. Let f_μ be an entire transcendental function $z \mapsto z\exp(z+\mu)$, where μ is a complex parameter. Put $f_\mu^n = f_\mu \circ f_\mu^{n-1}$ for a positive integer n, where f_μ^0 means the identity mapping of the complex plane $\boldsymbol{C}$. The Julia set J_μ of f_μ is defined as the set of all points on $\boldsymbol{C}$, in any neighbourhood of every point of which the sequence $\{f_\mu^n\}_{n=0}^\infty$ does not form a normal family.

Baker [1] proved the following theorem.

THEOREM. *There exists a real value of the parameter μ such that the Julia set J_μ of f_μ coincides with $\boldsymbol{C}$.*

Jang [3] proved the following result by studying Baker's argument in detail: There are infinitely many positive real values of μ with the property $J_\mu = \boldsymbol{C}$.

In this article, we study the distribution of values of μ stated in the above result of Jang. Noting another result $J_\mu \neq \boldsymbol{C}\,(-\infty < \mu < 2)$ of Jang [3], we restrict the parameter μ to the real value not less than 1.

1. Values μ_n and $\mu^{(n)}$ of the parameter μ. Obviously the set of singular values of $f: z \mapsto z\exp(z+\mu)$ consists of two values $z=0$ and $z=f_\mu(-1)$. The point $z=0$ is the only one finite transcendental singularity of the inverse function f_μ^{-1} of f_μ and this is fixed by f_μ. The point $z=f_\mu(-1)$ is the only one finite algebraic singularity of f_μ^{-1}.

For a fixed value μ of the parameter, we put

$$s_0(\mu) = -1 \quad \text{and} \quad s_n(\mu) = f_\mu(s_{n-1}(\mu)), \qquad n \geqq 1 .$$

The sequence $\{s_n(\mu)\}_{n=1}^\infty$ is the so-called orbit of the critical value $z=f_\mu(-1)$ of f_μ under the iteration of f_μ. The behaviour of this orbit plays a very important role in the study of the bifurcation of Julia sets J_μ. So, first we state some properties of $s_n(\mu)$.

Since the parameter μ is real, every $s_n(\mu)$ is negative and we have

$$s_n(\mu) = s_k(\mu)\exp\psi_{k,n-k}(\mu), \qquad 0 \leqq k \leqq n-1, \tag{1}$$

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