A CHARACTERIZATION OF THE MEAN CURVATURE
FUNCTIONS OF CODIMENSION-ONE FOLIATIONS

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Abstract. Walczak posed a problem on the characterization of the mean curvature
functions of codimension-one foliations. An affirmative answer to this problem is given
here. As an application, we get a simpler proof of the topological characterization, due
to the author, of codimension-one foliations consisting of constant mean curvature
hypersurfaces.

1. Introduction. Let $F$ be a transversely oriented codimension-one foliation of
a closed connected manifold $M$. If we choose a Riemannian metric $g$ on $M$, then we
have a smooth function $H$ on $M$, where $H(x)$ is the mean curvature at $x$ of the leaf $L_x$
of $F$ through $x$ with respect to the unit vector field $N$ which is orthogonal to $F$ and
whose direction coincides with the given transverse orientation. We call $H$ the mean
curvature function of $F$ with respect to $g$. In the previous papers [5], [6], the author
studied the following question posed by Walczak [8]: Which smooth function on $M$
can be written as the mean curvature function with respect to some Riemannian metric
on $M$? Such a smooth function on $M$ is said to be admissible. Some characterizations
of such functions are given in [5], [6]. However, it is not so easy to check whether the
given function is admissible or not by the characterizations given there.

On the other hand, Walczak also posed the following problem on the charac-
terization of admissible functions (see Langevin [3]):

**Problem.** Show that $f$ is admissible if and only if $f(x) > 0$ somewhere in any $N_{\text{max}}$
and $f(y) < 0$ somewhere in any $N_{\text{min}}$, where $N_{\text{max}}$ means a maximal Novikov component
and $N_{\text{min}}$ means a minimal Novikov component.

In this paper we study this problem. After reformulating the problem, we give an
affirmative answer to this problem in §3. As an application, we give in §4 a simpler
proof of the topological characterization in Oshikiri [6] of codimension-one foliations
consisting of constant mean curvature hypersurfaces.

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