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ALMOST PERIODIC SOLUTIONS OF A COMPETITION SYSTEM WITH DOMINATED INFINITE DELAYS

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Abstract. In this paper, we consider an n-species almost periodic Lotka-Volterra competition system with dominated infinite delays. By constructing suitable Lyapunov functionals, we are able to show that, under a set of algebraic conditions, the system has a unique positive almost periodic solution which is globally attractive.

1. Introduction. In this paper, we consider an almost periodic Lotka-Volterra system

$$\dot{x}_i(t)=x_i(t)\left[b_i(t)-\sum_{j=1}^{n} a_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)x_j(s)ds\right], \qquad i=1,\dots,n, \tag{1.1}$$

which describes a model of the dynamics of an n-species competition in mathematical ecology. When the system (1.1) has delay-independent dominated terms, it takes the form

$$\dot{x}_i(t)=x_i(t)\left[b_i(t)-a_{ii}(t)x_i(t)-\sum_{j=1,j\neq i}^{n} a_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)x_j(s)ds\right], \qquad i=1,\dots,n. \tag{1.2}$$

Recently, Gopalsamy [3] discussed the system (1.2) with ω-periodic coefficients b_i, a_{ij} $(i, j=1,\dots,n)$ and proved that, under a set of delay-independent algebraic conditions, the system (1.2) has a unique globally attractive ω-periodic solution. Murakami [10] generalized the discussion to the system (1.2) with almost periodic parameters b_i, a_{ij} $(i, j=1,\dots,n)$. By investigating the stability properties of the solutions of the system (1.2), Murakami [10] was able to show that (1.2) has an almost periodic solution. We also refer to Hamaya [7] and Hamaya and Yoshizawa [8] for further discussion on the periodic and almost periodic system (1.2), respectively. As one can see easily, when such delay-independent dominated terms are not present, the argument used in Gopalsamy [3], Hamaya [7], Hamaya and Yoshizawa [8] and Murakami [10] cannot be used for (1.1). For (1.1), when $n=1$, the related problem has been studied recently by Gopalsamy et al. [5] in the periodic case and Gopalsamy and He [4] and Seifert [11] in the almost periodic case. We also refer to He and Gopalsamy [9] for the

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