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NUMBER OF ZEROS OF SOLUTIONS TO SINGULAR INITIAL VALUE PROBLEMS

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Abstract. The behavior of solutions of singular initial value problems is studied for a second order ordinary differential equation. The main purpose of this paper is to obtain sharp sufficient conditions so that any solution has a finite number of zeros or infinitely many zeros. We treat them systematically and generalize previous results by using the Pohozaev identity. As an application, we investigate the number of zeros of radially symmetric solutions to generalized Laplace equations.

1. Introduction. The asymptotic behavior of solutions is one of the main topics in the theory of ordinary differential equations. In particular, the finiteness of the number of zeros of solutions is a fundamental question. In this paper we consider the behavior of solutions to an equation of the form

$$(\varphi(v_t))_t + k(t)f(v) = 0\,, \tag{1.1}$$

where

$$\varphi(\xi) = |\xi|^{m-1}\operatorname{sgn}\xi\,, \qquad m > 1\,.$$

Here, we introduce the following assumptions on $f(v)$:

$$(f.0)\qquad \begin{cases} f(v) \in C(\boldsymbol{R}) \cap C^1(\boldsymbol{R}\setminus\{0\})\,, \\ vf(v) > 0 \quad \text{for} \quad v \neq 0\,, \\ \displaystyle\limsup_{v\to 0} \frac{v|f'(v)|}{f(v)} < \infty\,, \end{cases}$$

$$(f.1)\qquad q_1 := \inf_{v\neq 0} \frac{vf'(v)}{f(v)} > m-1\,,$$

$$(f.2)\qquad q_2 := \sup_{v\neq 0} \frac{vf'(v)}{f(v)} < \infty\,.$$

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