CERTAIN ALGEBRAIC SURFACES OF GENERAL TYPE WITH IRREGULARITY ONE AND THEIR CANONICAL MAPPINGS

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Abstract. In this paper, we show the existence of certain algebraic surfaces of general type with irregularity one, and investigate the canonical mappings of these surfaces. Such a surface has a pencil of non-hyperelliptic curves of genus 3 over an elliptic curve, and is obtained as the minimal resolution of a relative quartic hypersurface with at most rational double points as singularities, of the projective plane bundle over an elliptic curve. We use some results on locally free sheaves over elliptic curves by Atiyah and Oda to prove the existence.

1. Introduction. Let $S$ be a minimal nonsingular projective surface defined over $\mathbb{C}$. $S$ is said to be canonical if the rational mapping $\Phi_{|K_S|}$ defined by the canonical linear system $|K_S|$ is birational.

In this paper, we show for all values of $p_g(S) \geq 2$ the existence of minimal algebraic surfaces of general type with $K_S^2 = 3p_g(S)$ and $q(S) = 1$, and study their canonical mappings. Note that the case $p_g(S) = 1$ was studied by Catanese and Ciliberto [7].

(I) (Castelnuovo-Horikawa’s inequality, cf. [5, Théorème 5.5], [12, Lemma 1.1]). If $S$ is a canonical surface, then

$$K_S^2 \geq 3p_g(S) - 7.$$ 

(II) Castelnuovo obtained canonical surfaces with $K_S^2 = 3p_g(S) - 7$ (cf. [6]). Such a surface $S$ satisfies $q(S) = 0$, and with a few exceptions $S$ is birational to a relative quartic hypersurface of a $\mathbb{P}^2$-bundle over $\mathbb{P}^1$ which has at most rational double points as singularities.

In general, a nonsingular relative quartic hypersurface in a $\mathbb{P}^2$-bundle over a nonsingular curve $C$ of genus $b$ satisfies

$$K_S^2 = 3p_g(S) + 7(b - 1), \quad q(S) = b.$$ 

We may ask whether a canonical surface $S$ satisfying these equalities is obtained as the minimal resolution of a relative quartic hypersurface with at most rational double points, of a $\mathbb{P}^2$-bundle over a nonsingular curve $C$ of genus $b$. Konno [15, Lemma 3.1, Theorem 3.2] proved that it is the case if $b = 1$. Namely, if $S$ is a canonical surface satisfying $K_S^2 = 3p_g(S)$ and $q(S) = 1$, then $S$ is the minimal resolution of a relative quartic