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MAXIMAL OPERATORS ASSOCIATED WITH COMMUTATORS OF SPHERICAL MEANS

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Abstract. In this paper, we prove that L^2 boundedness for the maximal operators associated with the commutators generated by BMO functions and some multiplier operators. And we also study the L^p boundedness for the maximal operator associated with the commutators of spherical means and a function in BMO or Lipschitz space.

1. Introduction. Coifman and Meyer observed that the L^p boundedness for the commutator $[b, T]$ defined by

$$[b, T]f(x)=b(x)Tf(x)-T(bf)(x)$$

could be obtained from the weighted L^p estimate for T with A_p weight when $b\in\mathrm{BMO}$ and T is a standard Calderón-Zygmund singular integral operator (see [4]), where A_p is the weight function class of Muckenhoupt (see [14, chapter V] for the definition and properties of A_p). In 1993, Alvarez, Babgy, Kurtz and Pérez [1] developed the idea of Coifman and Meyer, and established a general boundedness criterion for the commutators of linear operators. Their result can be stated as follows.

THEOREM A. *Let E be a Banach space, $1<p, q<\infty$. Suppose that the linear operator T: $C_0^\infty(\boldsymbol{R}^n)\to M(E)$ satisfies the weight estimates*

$$\|Tf\|_{L^p_w(E)}\le \bar{C}\|f\|_{p,w}$$

for all $w\in A_q$ and $\bar{C}$ depends only on n, p and $\tilde{C}_q(w)$ (the A_q constant of w), but not on the weight w. Then for any positive integer k and $b(x)\in\mathrm{BMO}(\boldsymbol{R}^n)$, the commutator

$$T_{b,k}f(x)=T((b(x)-b(\cdot))^k f)(x)$$

is bounded from $L^p_u(\boldsymbol{R}^n)$ to $L^p_u(E)$ for all $u\in A_q$ with norm $C(p, n, k, \tilde{C}_q(u))\|b\|^k_{\mathrm{BMO}}$.

This result is of great importance and is suitable for many classical operators in harmonic analysis. But for some important operators, the criterion of Alvarez-Babgy-Kurtz-Pérez breaks down. Let us consider the maximal operator of the spherical means defined by

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