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BOUNDS FOR THE ORDER OF AUTOMORPHISM GROUPS OF HYPERELLIPTIC FIBRATIONS

TATSUYA ARAKAWA

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Abstract. For a nonsingular complex algebraic surface with a pencil of hyperelliptic curves of genus g over a nonsingular algebraic curve, we take two approaches to get upper bounds for the order of its automorphism group as a generalization of Chen's results on genus two fibrations. If the genus of the base curve is neither one nor zero, we estimate the order of each automorphism group of the base curve and the general fiber by a theorem of Hurwitz and that of Tuji. In the cases of rational or elliptic base curves, we use the inequality of Horikawa-Persson to see the contribution of singular fibers.

1. Introduction. Let S be a nonsingular complex projective surface and C a nonsingular projective curve of genus π. Let $f:S\to C$ denote a relatively minimal fibration of curves of genus g.

An *automorphism* of f is, by definition, a pair of $\tilde{\sigma}\in \mathrm{Aut}(S)$ and $\sigma\in\mathrm{Aut}(C)$ which satisfies

$$f\tilde{\sigma}=\sigma f .$$

The group of automorphisms of f will be denoted by $\mathrm{Aut}(f)$. (cf. [3, Definition 0.1]).

Suppose S is a surface of general type and G a subgroup of $\mathrm{Aut}(f)$. Then Xiao [9] showed the following upper bounds for the order of G:

PROPOSITION 1.1 (cf. [9, Proposition 1]).

$$|G|\leq\begin{cases}882K_S^2 & \text{if } \pi\geq 2\\ 168(2g+1)(K_S^2+8g-8) & \text{otherwise}\,.\end{cases}$$

Furthermore Chen [3] obtained a more detailed estimate in the case of genus two fibrations:

PROPOSITION 1.2 (cf. [3, Theorem 0.1]). *Let $f: S\to C$ be a relatively minimal fibration of genus two. Then*

$$|G|\leq 504K_S^2$$

for $\pi\geq 2$. If f is not locally trivial, then

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