Abstract. We combine results of Giulini and Mauceri and our earlier work to obtain an almost-everywhere convergence result for the Bochner-Riesz means of the inverse spherical transform of bi-invariant $L^p$ functions on a noncompact rank one Riemannian symmetric space. Following a technique of Kanjin, we show that this result is sharp.

1. Notation. Suppose that $G/K$ is a noncompact rank one Riemannian symmetric space of dimension $d$. Here functions on $G/K$ can be viewed as being right-$K$-invariant functions on $G$, and $K$-invariant functions on $G/K$ are identified with bi-$K$-invariant functions on $G$. Denote by $-\Delta_0$ the Laplace-Beltrami operator on $G/K$, and $-\Delta$ its self-adjoint extension to $L^2(G/K)$. Its spectral resolution is

$$-\Delta = \int_{|\rho|^2}^{\infty} t dE(t),$$

where the constant $|\rho|^2$ depends on the geometry of $G/K$. For every $z \in \mathbb{C}$ with $\Re(z) \geq 0$ there are the Bochner-Riesz mean operators

$$S_{R}^z f = \int_{|\rho|^2}^{\infty} \left(1 - \frac{t}{R}\right)^z dE(t)f.$$

In fact there is a $C^\infty(K\backslash G/K)$ kernel $s^z_R$ so that

$$S_{R}^z f = f \ast s^z_R, \quad \text{for all } f \in C^\infty_c(G/K).$$

The special case $z = 0$ amounts to the usual partial sums:

$$S_{R}^0 f = E_R f, \quad \text{for } R \geq |\rho|^2,$$

and these converge in norm for elements $f \in L^2(G/K)$,

$$\|f - S_{R}^0 f\|_2 \to 0, \quad \text{as } R \to \infty.$$

1991 Mathematics Subject Classification. Primary 43A50; Secondary 33C55, 22E30, 43A90.

Key words and phrases. Rank-one symmetric space, Bochner-Riesz mean, spherical transform, complex interpolation, Cantor-Lebesgue theorem, maximal function.

CM partially supported by the CNR and MURG.

EP partially supported by the ICMS (International Centre for Mathematical Sciences).