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THE RIGIDITY FOR REAL HYPERSURFACES IN A COMPLEX PROJECTIVE SPACE

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Abstract. We prove a rigidity theorem for real hypersurfaces in a complex projective space of complex dimension $n \geq 4$. As an application of this rigidity theorem, we classify all intrinsically homogeneous real hypersurfaces in the complex projective space.

Introduction. Let $P_n(C)$ be an n-dimensional complex projective space. It is an open question whether a real hypersurface in $P_n(C)$ has rigidity or not. More precisely,

if M is a $(2n-1)$-dimensional Riemannian manifold and $\imath$, $\hat{\imath}$ are two isometric immersions of M into $P_n(C)$, then are $\imath$ and $\hat{\imath}$ congruent?

To this problem, many authors including the present ones gave some partial solutions (see [1], [3], [4] and [5]). Recall that an almost contact structure (ϕ, ξ, η) is naturally induced on a real hypersurface in $P_n(C)$ from the complex structure of $P_n(C)$, and ξ is called the *structure vector field*. The rank of the second fundamental tensor or the shape operator of a real hypersurface in $P_n(C)$ is said to be the *type number*. As one of the above-mentioned solutions, the following is known.

THEOREM A ([1]). *Let M be a $(2n-1)$-dimensional connected Riemannian manifold, and $\imath$ and $\hat{\imath}$ be two isometric immersions of M into $P_n(C)$ $(n \geq 3)$. If the two structure vector fields coincide up to sign on M and the type number of $(M, \imath)$ or $(M, \hat{\imath})$ is not equal to 2 at every point of M, then $\imath$ and $\hat{\imath}$ are rigid, that is, there exists an isometry φ of $P_n(C)$ such that $\varphi \circ \imath = \hat{\imath}$.*

The purpose of this paper is to give a solution of the rigidity problem using Theorem A. Namely, first of all we shall prove:

THEOREM 1. *Let M be a $(2n-1)$-dimensional Riemannian manifold, and $\imath$ and $\hat{\imath}$ be two isometric immersions of M into $P_n(C)$ $(n \geq 4)$. Then the two structure vector fields coincide up to sign on M.*

The following is immediate from Theorems A and 1.

THEOREM 2. *Let M be a $(2n-1)$-dimensional connected Riemannian manifold, and $\imath$ and $\hat{\imath}$ be two isometric immersions of M into $P_n(C)$ $(n \geq 4)$. If the type number of $(M, \imath)$ or $(M, \hat{\imath})$ is not equal to 2 at every point of M, then $\imath$ and $\hat{\imath}$ are rigid, that is, there exists*

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