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A BOUNDARY UNIQUENESS THEOREM FOR SOBOLEV FUNCTIONS

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Abstract. We show that under a condition on the Dirichlet integral, Sobolev functions with zero boundary values at the points of a set of positive capacity are identically zero.

1. Main result. All zeros of nonconstant analytic functions of the complex plane are isolated by a basic topological property of analytic functions. Results on the size of the zero set of the radial limits or nontangential boundary values of analytic functions of the unit disk and some related classes of functions have been proved by several authors. We now briefly review such theorems.

Beurling [1] proved that for nonconstant analytic functions f of the unit disk B with $|f'|\in L^2(B)$ the radial limit values cannot be equal to zero on a set of positive capacity. Carleson [2, Thm. 4] found examples of nonconstant analytic functions f of B with $|f'|\in L^2(B)$ and with radial limit zero on a given closed set $E\subset\partial B$ of zero capacity, and also proved other results about such functions. Tsuji [9] proved that an analytic function f, having radial limits zero on a set of positive capacity, is identically zero provided that there exists $c>0$ with

$$I(\varepsilon)=\int_{B_\varepsilon}|f'|^2dm\le c\varepsilon^2 \tag{1}$$

for all $\varepsilon\in(0, 1/2)$, where $B_\varepsilon=\{z\in B:|f(z)|<\varepsilon\}$ and dm is an element of the Lebesgue measure on B. Jenkins [4] extended this result assuming that the integral in (1) has order $\varepsilon^2\log(1/\varepsilon)$ (see also Villamor [11] with $o(\varepsilon^2\log(1/\varepsilon))$). Recently, Koskela [5] has established this result for $ACL^p(B)$-functions u with

$$I(\varepsilon)=\int_{B_\varepsilon}|\nabla u|^pdm\le C\varepsilon^p\left(\log\frac{1}{\varepsilon}\right)^{p-1}, \qquad 1<p\le n, \tag{2}$$

for $\varepsilon\in(0, 1/2)$, where $B=\{x\in R^n:|x|<1\}$ and

$$B_\varepsilon=\{x\in B:|u(x)|<\varepsilon\}, \qquad \varepsilon>0.$$

For related results see Mizuta [8].

Let $\mathscr{B}\subset R^n$ be a bounded domain and u be a (continuous) $ACL^p(\mathscr{B})$-function. Fix

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