BEST BOUNDS OF AUTOMORPHISM GROUPS OF HYPERELLIPTIC FIBRATIONS

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Abstract. For a relatively minimal hyperelliptic fibration, the best bounds of the orders of its automorphism group are obtained.

Let S be a smooth projective surface over the complex number field. A hyperelliptic fibration is a morphism $f: S \to C$ where C is a projective curve such that a general fiber of f is a smooth hyperelliptic curve.

DEFINITION 0.1. An automorphism of the fibration $f: S \to C$ is a pair of automorphisms $(\tilde{\alpha}, \alpha)$ with $\tilde{\alpha} \in \mathrm{Aut}(S)$, $\alpha \in \mathrm{Aut}(C)$ such that the diagram

$$
\begin{array}{ccc}
S & \xrightarrow{\ \tilde{\alpha}\ } & S \\
f \downarrow & & \downarrow f \\
C & \xrightarrow{\ \alpha\ } & C
\end{array}
$$

commutes.

The automorphism group of a fibration f will be denoted by $\mathrm{Aut}(f)$. Let G be a subgroup of $\mathrm{Aut}(f)$, G. Xiao has obtained upper bounds for the order of G:

PROPOSITION 0.1 ([6, Proposition 1]). *Suppose S is a complete surface of general type over the complex number field with a relatively minimal fibration $f: S \to C$ whose general fiber is of genus $g \geq 2$. Then*

$$
|G| \leq \begin{cases} 882 K_S^2 & \text{if } g(C) \geq 2 \\ 168(2g+1)(K_S^2 + 8g - 8) & \text{otherwise.} \end{cases}
$$

When $g = 2$, we have shown the following result.

THEOREM 0.1 ([3, Theorem 0.1]). *Suppose S is a complete surface of general type over the complex number field with a relatively minimal genus 2 fibration $f: S \to C$. Then*

$$
|G| \leq 504 K_S^2 .
$$