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CODIMENSION-ONE FOLIATIONS AND ORIENTED GRAPHS

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Abstract. In this paper, an oriented graph $G(M, F)$ is assigned to each codimension-one foliation (M, F), and topological relations between (M, F) and $G(M, F)$ are studied. A strong relation between admissible functions of (M, F) and $G(M, F)$ is given.

1. Introduction. Let (M, F) be a transversely oriented codimension-one foliation F of a closed oriented manifold M. On the set of all leaves of F, Novikov [6] introduced a partial order to define a so-called Novikov component. On the other hand, it is well-known that a partially ordered set is described as an oriented graph. In this paper, we assign to each (M, F), in a unique way, an oriented graph $G(M, F)$ by a similar way to Novikov's method, and show that for any oriented graph G, there is a codimension-one foliation (M, F) with $G=G(M, F)$. These are done in §3. We also show that there is a 'nice' embedding $\phi : G(M, F) \to M$, and in §4 we prove that the induced homomorphism $\phi_* : \pi_1(G(M, F)) \to \pi_1(M)$ is injective. Walczak [15] introduced the notion of admissible functions of (M, F) and the present author defined the notion of admissible functions of oriented graphs in [10]. As an application of the viewpoint obtained above, we show that these two notions of admissible functions are essentially same. This is done in §5. Finally, in §6, we give a brief discussion on Riemannian labels of oriented graphs, whose definition comes naturally from our viewpoint, and on the Laplacians on graphs.

2. Preliminaries. We begin this section with some definitions on graphs. For the definition of cellular complexes, see Spanier [13], and for generalities on graph theory, see Bollobas [2].

G is called a graph if G is a finite one-dimensional cellular complex. We set $V=V(G)=\{v_i\}=\{\text{all 0-cells of } G\}$ and $E=E(G)=\{e_a\}=\{\text{all 1-cells of } G\}$. We call each $v \in V(G)$ a vertex, and $e \in E(G)$ an edge. For $e \in E(G)$, we also set $V(e)=\mathrm{Cl}(e)-e=\{\text{endpoints of } e\}$, where the closure $\mathrm{Cl}(e)$ of e is taken in G.

REMARK. (a) $V(e)$ may consist of only one point $\{v\}$. In this case, we call e a loop at v.

(b) $V(e_a)=V(e_b)$ may occur even if $e_a \neq e_b$. In this case, G is called a multigraph (see Bollobas [2]).

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