

A CORRESPONDENCE BETWEEN THE MODULI SPACES OF VECTOR BUNDLES OVER A CURVE

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Abstract. After establishing a correspondence between a smooth moduli space of vector bundles on a curve and a self-product of the Jacobian, the nontriviality of the Griffiths group of the moduli space for a general curve is proved.

1. Introduction. Let \mathcal{N} denote the moduli space of isomorphism classes of stable vector bundles, over a compact Riemann surface X , of rank r and of a fixed determinant of degree d , with r and d being mutually coprime. Let $P(\mathcal{E})$ be the universal projective bundle over the Cartesian product $X \times \mathcal{N}$. The characteristic classes $a_k \in H^{2k}(X \times \mathcal{N}, \mathcal{Q})$, where $k = 2, 3, \dots, r$, of $P(\mathcal{E})$, give rise to homomorphisms, from $H_1(X, \mathcal{Q})$ to $H^{2k-1}(\mathcal{N}, \mathcal{Q})$, by the “slant product” operation. Since the homology algebra, $H_*(J(X), \mathcal{Q})$, of the Jacobian $J(X)$ of X is the exterior algebra $\bigwedge H_1(X, \mathcal{Q})$, combining all these homomorphisms given by the characteristic classes of $P(\mathcal{E})$, we get an algebra homomorphism from $H_*(J^{r-1}, \mathcal{Q})$ (the homology algebra of the $(r-1)$ -fold self-product of J) to the cohomology algebra $H^*(\mathcal{N}, \mathcal{Q})$. The details of this construction are given in Section 2.

We show that the above formally constructed homomorphism has a geometric origin. More precisely, there is a correspondence cycle on the product $\mathcal{N} \times J^{r-1}$, which is canonical as an element of the Chow ring (cycles modulo rational equivalence) of $\mathcal{N} \times J^{r-1}$, such that the above homomorphism is induced by this cycle (Theorem 2.5). As an application of this result, we construct nonzero elements of the Griffiths group of the moduli space \mathcal{N} for a general Riemann surface (Theorem 4.8). These elements are constructed from the nonzero elements of the Jacobian of a general curve discovered by Ceresa [C].

The above results extend to the more general context of smooth moduli spaces of parabolic bundles.

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2. Map between Hodge cycles. Let X be a compact Riemann surface, or equivalently, a connected smooth projective curve over \mathbb{C} . Assume that the genus $g := \text{genus}(X) \geq$