A CORRESPONDENCE BETWEEN THE MODULI SPACES
OF VECTOR BUNDLES OVER A CURVE

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Abstract.   After establishing a correspondence between a smooth moduli space of vec-
tor bundles on a curve and a self-product of the Jacobian, the nontriviality of the Griffiths
group of the moduli space for a general curve is proved.

1. Introduction. Let $\mathcal{N}$ denote the moduli space of isomorphism classes of stable
vector bundles, over a compact Riemann surface X, of rank r and of a fixed determinant of
degree d, with r and d being mutually coprime. Let $P(\mathcal{E})$ be the universal projective bundle
over the Cartesian product $X \times \mathcal{N}$. The characteristic classes $a_k \in H^{2k}(X \times \mathcal{N}, \mathbf{Q})$, where
$k = 2, 3, \ldots, r$, of $P(\mathcal{E})$, give rise to homomorphisms, from $H_1(X, \mathbf{Q})$ to $H^{2k-1}(\mathcal{N}, \mathbf{Q})$,
by the "slant product" operation. Since the homology algebra, $H_*(J(X), \mathbf{Q})$, of the Jacobian
$J(X)$ of X is the exterior algebra $\bigwedge H_1(X, \mathbf{Q})$, combining all these homomorphisms given by
the characteristic classes of $P(\mathcal{E})$, we get an algebra homomorphism from $H_*(J^{r-1}, \mathbf{Q})$ (the
homology algebra of the $(r-1)$-fold self-product of J) to the cohomology algebra $H^*(N, \mathbf{Q})$.
The details of this construction are given in Section 2.

We show that the above formally constructed homomorphism has a geometric origin.
More precisely, there is a correspondence cycle on the product $\mathcal{N} \times J^{r-1}$, which is canonical
as an element of the Chow ring (cycles modulo rational equivalence) of $\mathcal{N} \times J^{r-1}$, such
that the above homomorphism is induced by this cycle (Theorem 2.5). As an application of
this result, we construct nonzero elements of the Griffiths group of the moduli space $\mathcal{N}$ for
a general Riemann surface (Theorem 4.8). These elements are constructed from the nonzero
elements of the Jacobian of a general curve discovered by Ceresa [C].

The above results extend to the more general context of smooth moduli spaces of para-
bolic bundles.

2. Map between Hodge cycles. Let X be a compact Riemann surface, or equiva-
lently, a connected smooth projective curve over $\mathbf{C}$. Assume that the genus $g := \mathrm{genus}(X) \geq$