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COHOMOLOGY THEOREMS FOR ASYMPTOTIC SHEAVES

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Abstract. In this paper, we study the sheaves $\mathcal{A}_E^{<0}$ and $\mathcal{A}_E^{<-\kappa}$ of strongly asymptotically developable functions with null expansion, which are subsheaves of $\mathcal{A}$ defined by Majima. Following the method developed in one variable by Sibuya, and in several variables by Majima, we compute the first cohomology group of the n-torus and the boundary of the real blow-up with coefficients in these sheaves. The same technique is used to study the multiplicative case (sheaves of non-abelian groups), in order to calculate the first cohomology set. This generalizes previous results of Majima, Haraoka and Zurro.

1. Definitions and notations.

A polysector $V = V_1 \times \cdots \times V_n$ in $\boldsymbol{C}^n$ is a product of open sectors, an open sector being a set of the type

$$V_{\alpha,\beta,R} = \left\{z \in \boldsymbol{C} \mid \alpha < \arg z < \beta, 0 < |z| < R\right\},$$

where $R \in (0, \infty]$. The number $\beta - \alpha$ is the opening of $V_{\alpha,\beta,R}$. A subpolysector W of V ($W < V$) is $W_1 \times \cdots \times W_n$, where W_i is a closed sector of finite radius and strictly smaller opening than V_i.

$\mathcal{A}(V)$ will denote the $\boldsymbol{C}$-algebra of functions that are strongly asymptotically developable in V, introduced by Majima in [M1]. Let us recall that $f \in \mathcal{A}(V)$ if and only if there exist a family of functions

$$\mathcal{F} = \left\{f_{\alpha_J}(z_{J^c}) \in \mathcal{O}(V_{J^c}) \mid \emptyset \neq J \subseteq \{1, \dots, n\}, \alpha_J \in \boldsymbol{N}^J\right\}$$

such that, if $W < V$ and $N \in \boldsymbol{N}^n$, there exists $C_{W,N} > 0$ with

$$|f(z) - \mathrm{App}_N(\mathcal{F})(z)| < C_{W,N} \cdot |z^N| \text{ in } W,$$

where

$$\mathrm{App}_N(\mathcal{F})(z) = \sum_{\emptyset\neq J\subseteq\{1,\dots,n\}} \sum_{j\in J} \sum_{\alpha_J<N_J} (-1)^{\sharp J+1} \cdot f_{\alpha_J}(z_{J^c}) \cdot z_J^{\alpha_J}$$

and $J^c = \{1, \dots, n\} \setminus J$. We have used the following notations: if $J \in \{1, \dots, n\}$, $V_J := \prod_{j\in J} V_j$, and z_J is the element of V_J obtained by projection of $z \in V$ to V_J. This family $\mathcal{F}$ (the *total family of coefficients of* f) is unique, and it will be denoted by $TA(f)$. As in [M1], for $f \in \mathcal{A}(V)$, $\emptyset \neq J \subseteq \{1, \dots, n\}$, $FA_J(f)$ will denote the series

$$FA_J(f) = \sum_{\alpha_J\in\boldsymbol{N}^J} f_{\alpha_J}(z_{J^c}) z_J^{\alpha_J} \in \mathcal{A}(V_{J^c})[[z_J]].$$

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