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CORRECTION: A NOTE ON THE FACTORIZATION THEOREM OF TORIC BIRATIONAL MAPS AFTER MORELLI AND ITS TOROIDAL EXTENSION

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This erratum describes:

1. The failure of the algorithm in [AMR] and [Morelli1] for the strong factorization pointed out by Kalle Karu,

2. The statement of a refined weak factorization theorem for toroidal birational morphisms in [AMR], in the form utilized in [AKMR] for the proof of the weak factorization theorem for general birational maps, avoiding the use of the above mentioned algorithm for the strong factorization, and

3. A list of corrections for a few other mistakes in [AMR], mostly pointed out by Laurent Bonavero.

We would like to emphasize that though [AMR] is a joint work with D. Abramovich and S. Rashid, the author of this erratum, Kenji Matsuki, is solely responsible for all the errors above.

1. K. Karu pointed out that the procedure in Proposition 7.8 in [AMR] does not preserve the condition $(*)$, contrary to its assertion, and thus the proof does not work. Moreover, the entire algorithm for the strong factorization in Section 7 of [AMR] based upon Proposition 7.8, which attempted to correct the logic of the line of argument of the original one described in [Morelli1] but is identical to it as an algorithm, does not work, as shown by the following example of a cobordism representing a toric birational map in dimension 3:

Consider a 4-dimensional simplicial cobordism Σ describing three smooth star subdivisions of the 3-dimensional cone $\langle v_1, v_2, v_3\rangle$ at $\langle v_1 + v_2\rangle$, $\langle v_2 + v_3\rangle$ and $\langle v_1 + v_2 + v_3\rangle$, in this order. The four maximal cones in this cobordism are all pointing up, but the new cobordism Σ', obtained by the procedure of Proposition 7.8 in [AMR], contains a cone that is not pointing up. Indeed, each of the four given maximal cones has exactly one positive extremal ray and one extremal ray in the link of its circuit. The algorithm subdivides the cobordism at the barycenters of the 2-dimensional cones generated by the positive and the link extremal rays. After subdividing the two topmost cones, one of the new cones will be a pointing up cone with one positive extremal ray $\langle \rho\rangle$ and three negative extremal rays $\langle \rho_{12}\rangle$, $\langle \rho_{23}\rangle$ and $\langle \rho_3\rangle$ with $n(\pi(\rho_{12})) = v_1 + v_2$, $n(\pi(\rho_{23})) = v_2 + v_3$ and $n(\pi(\rho_3)) = v_3$. The next two subdivisions are at the midrays (barycenters) $\zeta_1 = \mathrm{Mid}(\langle \rho_{12}, \rho_{23}\rangle, l_{r(\langle \rho_{12},\rho_{23}\rangle)})$ and $\zeta_2 = \mathrm{Mid}(\langle \rho_{12}, \rho_3\rangle, l_{r(\langle \rho_{12},\rho_3\rangle)})$. The resulting subdivision Σ' contains a cone $\langle \rho, \rho_{12}, \zeta_1, \zeta_2\rangle$,