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ON THE EQUIVALENCE OF FOUR CONVERGENCE THEOREMS FOR THE AP-INTEGRAL

1. Introduction

Liao and Chew gave the controlled convergence theorem for the AP-integral based on $UACG^*_{ap}$, and two corollaries involving the "Uniformly approximate strong Lusin condition ($UASL$, for short)" ([LC], Theorem 4.1, Corollary 4.1 and 4.2). The term "generalized P-Cauchy" was introduced by R. Gordon. He proved that $\{f_n\}$ is uniformly Henstock integrable on $[a,b]$ iff $\{F_n\}$ is generalized P-Cauchy on $[a,b]$, where F_n is the primitives of f_n ([G], Theorem 5). The purpose of this paper is to prove the two conditions given by Theorem 4.1 and Corollary 4.1 in [LC], and the condition "generalized approximate P-Cauchy (GAP-Cauchy, for short)" which is an extension of the term generalized P-Cauchy, are equivalent to a weak uniformly AP-integrability condition (Theorem 4.1 and 4.2). Furthermore, all the above-mentioned conditions don't require the hypothesis that "$\{F_n\}$ converges everywhere on $[a,b]$".

2. A Review of Basic Definitions.

We will assume the reader to be familiar with the AP-integral. Throughout this paper, $[a,b]$ denotes a fixed finite closed interval. All functions are real-valued and measurable. The sets involved are always assumed to be measurable and we denote the measure by $|\cdot|$.

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