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# CERTAIN MEASURE ZERO, FIRST CATEGORY SETS

## I

P. Komjáth [4] constructed a measure zero, first category set containing a translated copy of every countable set of reals. Assuming Martin's Axiom, he generalized this by showing that a similar set exists containing a translated copy of every set of size smaller than $|R|$. In this paper we obtain some results exploring such sets. For example, we show that if a set $A$ contains a translated copy of every countable set, then its intersection with every nonempty open interval is of size $|R|$. The converse of this result is not true.

**Theorem 1.** *Let $A$ be a set of reals which contains a translated copy of every countable set of reals. If $F$ is a measure zero or first category additive subgroup of the reals $R$, or if $F$ is a set of size smaller than $|R|$, then $A \setminus F$ contains a translated copy of every countable set of reals.*

**Proof.** Let $N$ be a countable set of reals and let

$$X = \{x \in R : x + N \subseteq A\}, \quad \text{where } x + N = \{x + y : y \in N\}.$$

Suppose $(x + N) \cap F \neq \emptyset$ for every $x \in X$. Then $X \subseteq F - N$ and $X - X$ is contained in a countable union of translated copies of $F$ or $|X - X| < |R|$. Let $y \in R$ and $y \notin X - X$. The hypothesis implies that $r + ((N + y) \cup N) \subseteq A$ for some $r \in R$. Since $r + y + N \subseteq A$ and $r + N \subseteq A$, by definition of $X$, $r + y \in X$ and $r \in X$. Hence $y \in X - X$, which contradicts the choice of $y$. Thus $(x + N) \cap F = \emptyset$ for some $x \in X$ and this completes the proof.

**Corollary 1.** *If a set $A$ contains a translated copy of every countable set, then its intersection with every nonempty open interval is of size $|R|$.*

**Proof.** If $|A \cap I| < |R|$ for some nonempty open interval $I$, then by Theorem 1, $A \setminus (A \cap I)$ contains a translated copy of every countable set, and consequently