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Some interpolation problems in real and harmonic analysis[1]

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Suppose we have any function space F and a subspace G of "good" functions. For arbitrary $f \in F$, we wish to find $g \in G$ which coincides with f on some set E. This is an interpolation problem. It is necessary to distinguish between two variants of the problem.

In the first case, *interpolation with fixed knots*, a set E (not necessarily finite) is given a priori. If the problem is resolvable for every $f \in F$ we say E is an *interpolating set* for the pair (F, G).

In the second case, E is not given and it can be chosen, depending on f, in such a way that it may be "thick" in a metric sense or in cardinality. This is *free interpolation.* An elementary example of the first problem is polynomial interpolation by the Lagrange or Newton method. Another and deeper example is given in the famous Rudin-Carleson theorem on the disc-algebra of functions.

An important example of the second type of problem is the famous Menshoff "correction" theorem in Fourier analysis. In what follows I will be concerned with some aspects of interpolation of continuous functions arising in classical and harmonic analysis and I will describe recent progress and some open questions.

I. Interpolation by smooth functions.

Here we deal with the following problem: to what degree can one improve the smoothness of given function $f: I = [0, 1] \to \Re$ by free interpolation on perfect (nonempty) sets.

The history of this question begins with a curiosity. In the mid-thirties Ulam conjectured that for every $f \in C(I)$ one can define an analytic function g which coincides with f on a perfect set.

One might observe in favor of this conjecture that if f is "bad", say nowhere differentiable, or it has no interval of monotonicity, then some level sets of f are uncountable, so we can put $g \equiv const$.

[1] This is a summary of a talk given at the Fourteen Summer Symposium on Real Analysis at San Bernardino.