P.Muldowney, Magee College, University of Ulster, Northern Ireland

INFINITE-DIMENSIONAL GENERALISED RIEMANN INTEGRALS

In one dimension, we take intervals I of the form

$$]-\infty, a[, \ [u, v[, \ [b, \infty[$$

and define $|I|$ as

$$a, \ v - u, \ b$$

respectively. The pair (I, x) are associated if x is

$$-\infty, \ u \text{ or } v, \ +\infty$$

respectively. A gauge is a positive function $\delta(x)$ defined for $x \in R^* = R \cup \{-\infty, \infty\}$.
If I, x are associated then (I, x) or, simply, I, is δ-fine if $|I|$

$$< -1/\delta(x), \ < \delta(x), \ > 1/\delta(x)$$

respectively.

In n dimensions, an interval I is $I_1 \times \ \ldots \ \times I_n$, where each I_j is an interval of R. $|I|$ is the product of the $|I_j|$. If $x = (x_1, ..., x_n)$, $x_j \in R^*$, then (I, x) is an associated couple if (I_j, x_j) are associated in R, $1 \leq j \leq n$. Given $\delta(x) > 0$, I and (I, x) are δ-fine if I_j is δ-fine, $1 \leq j \leq n$. A partition of R^n is a finite collection of disjoint intervals I whose union is R^n. A division of R^n is a finite collection of associated (I, x) such that the intervals I form a partition of R^n. A division $\mathfrak{E}$ of R^n is δ-fine if each $(I, x) \in \mathfrak{E}$ is δ-fine.

Given a point function h(x), the Riemann sum is denoted by

$$(\mathfrak{E})\Sigma h(x)|I|,$$

or simply $\Sigma h(x)|I|$, where we take h(x) equal to zero by definition if x is at infinity.

Similarly, if h(I, x) is a function of associated (I, x), the Riemann sum is $\Sigma h(I, x)$, making a similar provision whenever x has an infinite component.

For the infinite dimensional case, let $]\tau', \tau[$ be an interval of R, and let $T = R^{]\tau', \tau[}$. Thus T is the set

$$\{x : t \rightarrow x(t), t \in \ ]\tau', \tau[, x(t) \in R\}.$$

T* is T with points at infinity adjoined, i.e. replace R by R* in previous definition.
Let $N = \{t_1, \ldots, t_n\}$, $x_j = x(t_j)$ for $1 \leq j \leq n$, $x(N) = (x_1, \ldots, x_n)$,
$I = I[N] = \{x : x(t_j) \in I_j, 1 \leq j \leq n\}$, $I(N) = I_1 \times \ \ldots \ \times I_n$, $|I| = |I[N]| = |I_1| . \ldots . |I_n|$.
(I[N], x) are associated in T if (I(N), x(N)) are associated in R^n. The definitions of partition and division of T follow those given for R^n.