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REMARKS ON UNIFYING PRINCIPLES IN REAL ANALYSIS

1. Unifying principles for proving fundamental theorems in real analysis

Several unifying principles for proving fundamental theorems in real analysis have been formulated in the mathematical literature. Such a principle is the principle of induction in continuum (cf. [7], [9], [10]). Some analogous principles are contained in [3], [4], [5], [8], and [11]. In this part of the paper we shall investigate the principles formulated in [8] and [11]. In particular the principle from [8] seems to be a very effective tool for simplifying proofs of some fundamental theorems in analysis. (See the second part of this paper.)

We shall formulate the principles from [8] and [11] for a chain (totally ordered set) $(X,<)$ having minimal element $(= a)$ and maximal element $(= b)$ and we shall suppose that $(X,<)$ has no gaps (i.e. for each two elements $x, y \in X$ with $x < y$ there exists a $z \in X$ such that $x < z$, $z < y$). In what follows we denote the interval topology on X by T.

The following properties (P_1) and (P_2) correspond to the principles from [8] and [11] respectively.

The chain $(X,<)$ is said to have the property (P_1) provided $<$ is the unique relation L on X satisfying the following conditions:

(A1) L is transitive, i.e. if $x\,L\,y$ and $y\,L\,z$, then $x\,L\,z$;

(A2) $L \subset <$;

(A3) L is locally valid, i.e. if $p \in X$, then there exists a neighborhood $V(p) \in T$ of the point p such that

$$x \in V(p),\ x < p \ \Rightarrow\ x\,L\,p\ , \quad \text{and}$$

$$x \in V(p),\ p < x \ \Rightarrow\ p\,L\,x\ .$$

The chain $(X,<)$ is said to have the property (P_2) provided each system S of closed intervals $[c,d] \subset X$ satisfying the conditions (B1) and (B2) contains $[a,b]$ $(= X)$, where