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THE SQUEEZING THEOREM IS INDEPENDENT

In [1] a system of six properties which completely characterizes the concept of convergence of real sequences was introduced. It consists of the following:

1.Definition *The collection of all real sequences will be denoted by S. The triple $(A, S, \equiv)$ is a convergence system on S provided that $A \subset S$, "$\equiv$" is a relation on A*[1] *and:*

A1. *If $X = \{x_n\}_{n=1}^{\infty} \in A$ and $k \in \mathbb{R}$, then $kX = \{k\,x_n\}_{n=1}^{\infty} \in A$*
A2. *If $X \equiv Y, Z \equiv W$, and $X + Z, Y + W \in A$, then $Y + W \equiv X + Z$.*
A3. *If $X, Y \in A$, $Z \in S$, $x_n \leq z_n \leq y_n$ for all n and $X \equiv Y$, then $Z \equiv X$.*
A4. *If $X \in A$ and Y, Z are subsequences of X, then $Y \equiv Z$.*
A5. $\{(-1)^n\}_{n=1}^{\infty} \notin A$.
A6. *If $X \notin A$ is bounded, then X has subsequences $Y, Z \in A$ with $Y \not\equiv Z$.*

In [1] it was shown that the only convergence system on S is $(C, S, \equiv_0)$ where C is the family of all sequences converging in the classical sense (i.e. in the Euclidian topology) and $X \equiv_0 Y$ iff $\lim_{n\to\infty} x_n - y_n = 0$.

In the same paper it was asked whether Property A3 (the Squeezing Theorem) is independent of the other five properties. We will give the likely affirmative answer by providing an elementary construction of a system $(A, S, \equiv)$ fulfilling A1, A2, A4, A5, and A6 but such that $A \neq C$. According to the just mentioned result $(A, S, \equiv)$ can not satisfy A3.

2.Definition *Denote by A the system of all sequences $\{x_k\}_{k=1}^{\infty}$ of the*

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[1] In [1] "$\equiv$" is a relation on S but it is interesting only on A.