Queries Editor: Richard J. O'Malley, Department of Mathematical Sciences, University of Wisconsin, Milwaukee, WI 53201

Editor's note: Query 7 of vol.16 No.1 1990-91 page 376 has been solved by Professors Paul Humke and Chris Freiling. The solution is in this volume of the R.E.A.. They have the editor's gratitude and the check is in the mail.

As was mentioned in the report on the Summer Symposium, a very lively problem session was held late Tuesday. The following is a selection of some of the problems and their solutions.

I. Problem (M. Laczkovich). Consider the following properties of a set $B \subset \mathbf{R}$.

(i) B cannot be covered by an F_σ set of measure zero.

(ii) There exists a non-empty closed set $F \subset \mathbf{R}$ such that every portion of F is of positive measure, and $F \cap B$ is residual in F.

It is easy to see that (ii)$\Longrightarrow$(i) holds for every $B \subset \mathbf{R}$. Is it true that (i)$\Longrightarrow$(ii) holds for every Borel set $B \subset \mathbf{R}$?

II. Problem 2 (J. S. Lipiński): Does there exist a non-bounded set $E \subset \mathbf{R}$ such that any real bounded function defined on E can be extended to a differentiable periodic function defined on the whole real line?

Solution (P.D. Humke and M. Laczkovich). The answer is affirmative. Let c and δ_n be positive numbers such that $\sum_{n=1}^{\infty} \delta_n = c$, and let t_n be an increasing sequence of positive numbers such that $t_{n+1}/t_n \geq (c+\delta_{n+2})/\delta_{n+1}$ $(n = 1, 2, \ldots)$. J. Lipiński proved that every bounded function defined on $E = \{t_n\}$ can be extended to a periodic and locally Lipschitz function defined on $\mathbf{R}$ (*Real Analysis Exchange* 7 (1) (1981-82), 129-134). More exactly, Lipiński showed that if ϕ is a bounded function on E, $\gamma = c - \delta_1/2$, and if $\psi : [0, c] \to \mathbf{R}$ is such that $\psi(0) = \psi(c)$ and $\psi^{-1}(\{\phi(t_n)\}) \cap [0, \gamma]$ contains an interval of length δ_{n+1} for every n, then there is a positive number r and a periodic extension f of ϕ such that the period of f is rc and $f(x) = \psi(x/r)$ $(x \in [0, rc])$.

We shall prove that ψ can be chosen to be differentiable; this will prove the existence of a differentiable, periodic extension.