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ON LOCALLY SYMMETRIC AND LOCALLY JENSEN FUNCTIONS

The present paper deals with finite real functions of a real variable. The notion of local symmetry has been introduced by M. Foran in her paper [F].

Definition 1. A function $f : R \to R$ (R - the real line) is said to be locally symmetric at a point x if there exists $\delta = \delta(x) > 0$ such that

$$(S) \qquad f(x+h) = f(x-h)$$

holds for every, h, $0 < h < \delta$. A function $f : R \to R$ is said to be locally symmetric, if it is locally symmetric at each $x \in R$.

The structure of locally symmetric functions is known. Every such a function is constant with the exception at most of a set, the closure of which is countable (see [D], [R], [T]). In his survey article [M] S. Marcus has introduced the notion of the uniform local symmetry.

Definition 2. A function $f : R \to R$ is said to be uniformly locally symmetric on a set A, if there exists $\delta > 0$ such that (S) holds for each h, $0 < h < \delta$, and for each $x \in A$.

The locally symmetric functions are functions for which the first symmetric difference $\Delta f(x,h) = f(x+h) - f(x-h)$ locally fulfils the equality $\Delta f(x,h) = 0$. We shall also deal with functions locally fulfilling the equality $\Delta^2 f(x,h) = 0$, where $\Delta^2 f(x,h) = f(x+h) + f(x-h) - 2f(x)$ is the second symmetric difference of f at x. In the literature such functions are known under the name locally Jensen functions (see [K]).

Definition 3. A function $f : R \to R$ is said to be locally Jensen at $x \in R$ if there exists $\delta = \delta(x) > 0$ such that

$$(J) \qquad \frac{1}{2}(f(x+h) + f(x-h)) = f(x)$$