Real Analysis Exchange Vol 15 (1989-90)

L. Zajíček, Charles University, Sokolovská 83, 186 00 Praha 8, Czechoslovakia

UNPUBLISHED RESULTS OF K. PEKÁR AND H. ZLONICKÁ ON PREPONDERANT DERIVATIVES AND M_4 - SETS

These results were obtained in 1981 thesis of K. Pekár [8] and in a 1982 student work of H. Zlonická (now Mrs. H. Palovská) written under the direction of the author of this note. They seem to be of some interest but, for various reasons, were not published.

1 The results of K. Pekár on preponderant derivatives

Pekár in his thesis answered some questions mentioned in [5] and [1]. We shall start with the definitions of notions involving preponderance.

The symbols λ, λ_e and λ_i will stand for the Lebesgue, the outer Lebesgue and the inner Lebesgue measure, respectively.

<u>**Definition 1.**</u> We shall say that $M \subset R$ is preponderant at $a \in R$ if $\liminf_{h\to 0+} \frac{\lambda_i(M\cap(a,a+h))}{h} > 1/2$ and $\liminf_{h\to 0+} \frac{\lambda_i(M\cap(a-h,a))}{h} > 1/2$. We say that $E \subset R$ is weakly preponderant at a if $\frac{\lambda_i(E\cap I)}{\lambda(I)} > 1/2$ for all sufficiently small intervals I containing a.

<u>**Definition 2.**</u> Let f be a real function, $a \in R$ and $A \in R \cup \{-\infty, \infty\}$. We say that

(i) (s)-lim $\mathrm{pr}_{x\to a} f(x) = A$ if there is a set E preponderant at a such that $\lim_{x\to a, x\in E} f(x) = A$,

(ii) lim $\mathrm{pr}_{x\to a} f(x) = A$ if $\{x : f(x) > r\}$ ($\{x : f(x) < s\}$) is preponderant at a for each $r < A$ ($s > A$),