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SOME SYMMETRIC COVERING LEMMAS

The first significant investigation of the symmetric derivative was that of Khintchine [9] in 1927. As well as obtaining some elementary properties by elementary and unoriginal means he introduces the first interesting and new technique into the study in order to obtain the fact that a measurable function with a symmetric derivative on a set is differentiable almost everywhere on that set. Basically his argument is that if there is a uniform estimate on the symmetric difference quotient of a function f on a set E then at points that are both density points of E and points of approximate continuity of f a similar estimate for the ordinary difference quotients is available. This kind of an argument has been repeated often in subsequent years. For example in Stein and Zygmund [16, p. 266] this method is used to prove that measurable functions are continuous at almost every point of measurable sets on which they are symmetrically continuous; while they do not acknowledge Khintchine specifically as a source of the techniques they use, the ideas are easily traced to him and they suggest that the result had been known for some time.

A basic ingredient of the Khintchine proof is the use of points of approximate continuity and so the technique applies primarily to measurable functions. Thus, for example, something new is needed in order to investigate the points of symmetric continuity of a function that is not given *a priori* to be measurable. For a function f that is *everywhere* symmetrically continuous classical methods can still be made to work. Let $\omega_f(x)$ be the oscillation of f at x. Then ω_f is symmetrically continuous too and it is measurable. Thus the Stein-Zygmund theorem shows that it must be a.e. continuous. Together with a theorem of Fried [8] this shows that f is a.e. continuous too. This argument is due to David Preiss and can be considered to replace that in [11].

More recently, no doubt motivated by the arguments in [9] and [11], Uher [20] with some considerable technical skill has refined these techniques