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WEIGHTED SYMMETRIC FUNCTIONS

In [11], I studied the classes of weighted symmetric functions, which are a generalization of the classes of symmetric and symmetrically continuous functions. By definition, we call a weight system of order n a set of real numbers

$$W_n = \{w_{-n}, \ldots, w_{-1}, w_0, w_1, \ldots, w_n\}$$

such that $\sum_{k=-n}^{n} w_k = 0$ and $|w_n|+|w_{-n}|>0$.

We say a weight system is even if

$$w_{-k} = w_k \qquad k = 0,1,\ldots,n$$

with $\sum_{k=1}^{n} w_k \neq 0$; then $w_0 = -2\sum_{k=1}^{n} w_k \neq 0$

and a weight system is odd if

$$w_{-k} = -w_k \qquad k = 0,1,\ldots,n$$

with $\sum_{k=1}^{n} w_k \neq 0$; then $w_0 = 0$.

We call symmetric difference with respect to a weight system W_n of order n for a finite real-valued function f(x) the following expression

$$\Delta f(x;W_n,h) = \sum_{k=-n}^{n} w_k f(x+kh/2).$$

A finite real-valued function f is said to be symmetric with respect to a weight system W_n if

$$\lim \Delta f(x;W_n,h) = 0, \qquad h\to 0.$$

In this paper, we generalize the properties of measurable symmetric functions proved by Mazurkiewicz [7], H. Auerbach [2], and C. J. Neugebauer [8] to functions