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SPECTRAL RADIUS OF NONSINGULAR TRANSFORMATIONS

We say that T is an invertible, nonsingular, ergodic transformation on a probability space $(X, \mathcal{B}, \mu)$ if $T:X\to X$ is one to one, $T(A)$ and $T^{-1}(A) \in \mathcal{B}$ whenever $A \in \mathcal{B}$, $\mu(T(A)) > 0$ and $\mu(T^{-1}(A)) > 0$ whenever $\mu(A) > 0$, and $\mu(A) = 0$ or 1 whenever $T(A) = A$. Flytzanis [1] introduced the spectral radius as an invariant for such transformations as follows: For every $A \in \mathcal{B}$ with $\mu(A) > 0$, let $r(T,A)$ denote the radius of convergence of the power series $\sum_{n=0}^{\infty} \mu(\Delta A_n)\, x^n$ where $A_n = \bigcup_{j=-n}^{n} T^j A$, $A_0 = A$, $A_{-1} = \varnothing$, and $\Delta A_n = A_n - A_{n-1}$. The spectral radius $r(T)$ is then equal to $\inf\{ r(T,A) : \mu(A) > 0\}$. It is clear that $r(T) \geq 1$, and if T is a periodic transformation (T^p = identity for some $p \geq 1$), then $r(T) = \infty$. We will assume that μ is nonatomic, so that, since T is ergodic, it can not be periodic. The purpose of this note is to prove: