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BOLYAI-GERWIEN THEOREM AND HILBERT'S THIRD PROBLEM

This talk is meant to be a supplement of Paul Humke's lecture on Laczkovich's solution to the well-known circle-squaring problem of Tarski.

Laczkovich's solution states that a disc and a square with the same area are translation-equidecomposable. On the other hand, when the pieces in the decomposition are restricted to be polygons we are going to see that even two polygons with the area same are not necessarily translation-equidecomposable. To this end, let D denote the group of all the rigid motions of the Euclidean plane. For any subgroup G of D, two polygons A and B are said to be G-equidecomposable if there exist a positive integer n, n elements $g_1, g_2, \cdots, g_n \in G$, n non-overlapping polygons $A_1, A_2, A_3, \cdots, A_n$, and n non-overlapping polygons $B_1, B_2, B_3, \cdots, B_n$ such that
$A = \cup A_i$, $B = \cup B_i$ and $g_i(A_i) = B_i$ for $i = 1, 2, 3, \cdots, n$. Then the well-known Bolyai-Gerwien Theorem states that any two polygons of equal area are D-equidecomposable. Improving this result, Hadwiger and Glur have proved the following theorem, where S is the subgroup of D consisting of all the translations and all the center inversions.

<u>Theorem 1</u>. (Hadwiger-Glur). Any two polygons with the same area