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Some Hausdorff variants of absolute continuity, Banach's condition(S) and Lusin's condition(N)

The old concept of Hausdorff dimensions has been given a revived interest due to its usefulness in the recent development of non-linear dynamic theory and fractal geometry. Thus, it seems worthwhile to use the concept of Hausdorff measures to consider some natural variants of the well-known concepts of the absolute continuity, the Banach's condition (S) and the Lusin's condition (N). To take into account some of the recent works by Foran [2], Iseki [3] and Ene [1], some variants using notions closely related to the Hausdorff measures are also considered. It should be noted, however, that thorough investigations of such variants and their possible applications to dynamic theory and fractal geometry still remain to be done.

First, let us make precise the notions that are closely related to Hausdorff measures. Let E be a set of real numbers. For such positive integer n, and for such positive real number β, let

$$\lambda_n^{\beta}(E) = \inf\{ \sum_{i=1}^{n} |I_i|^{\beta} : \ \langle I_i \rangle_{i=1}^{n} \text{ is a sequence of n open intervals which covers E}\},$$

and also let