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Differentiable Restrictions of Real Functions

We review some of the known theorems about differentiable and monotonic restrictions of continuous and arbitrary real functions and present some new results of this type. We adopt the convention that differentiable functions are assumed to have finite valued derivatives, and when we mean differentiability in the extended sense (allowing $f'(x) = +\infty$ or $-\infty$), we put quotation marks on "differentiability".

The first result of the type we are conisdering would be the following.

Theorem 1: For every continuous $f : [0,1] \to R$, there exists a perfect subset P of [0,1] such that $f|P$ is differentiable.

We don't know when this result was first discovered, but it certainly follows from Lebesgue's Theorem together with the monotonicity results of Minakshisundaram [13], Padmavally [15], Marcus [12], and Garg [7]. The set P in the conclusion of Theorem 1 cannot be made to have positive measure because of the existence of nowhere approximately differentiable continuous functions (see the paper of Jarnik [9] or [3, Ch. 16]).

In order to improve the conclusion of Theorem 1 (i.e. to obtain that $f|P$ is C^1 or twice differentiable or better), it is clear that one needs similar theorems for functions with