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THE HAUSDORFF DIMENSION OF VERY STRONGLY POROUS SETS IN R^n

For $E \subset R^n$, $n \geq 2$, $x \in E$, and $0 < r < \infty$, denote

$$p(E,x,r) = \sup\{s: B(y,s) \subset B(x,r) \setminus E \text{ for some } y\}.$$

Here $B(x,r)$ is the closed ball with centre x and radius r. It is easy to see by simple covering arguments that if $0 < p < 1$ and

$$\liminf_{r \downarrow 0} 2p(E,x,r)/r \geq p \quad \text{for } x \in E,$$

then the Hausdorff dimension $\dim E \leq d(p)$, where $d(p)$, $n-1 \leq d(p) < n$, is a constant depending only on n and p. But finding the best possible $d(p)$ seems to be a rather difficult problem. However it is possible to choose $d(p)$ in the following asymptotically precise way as p tends to 1:

THEOREM 1. $\lim_{p \uparrow 1} d(p) = n-1$. *In particular, if* $\lim_{r \downarrow 0} 2p(E,x,r)/r = 1$ *for* $x \in E$, *then* $\dim E \leq n-1$.

This result is an easy corollary to the following density theorem for the s-dimensional Hausdorff measure H^s proved in [M]:

THEOREM 2. *Let* $n-1 < s < n$. *For any* $b > 0$ *there is* $c(b) > 0$, *depending also on* n *and* s, *with the following property: Suppose* E *is an* H^s *measurable subset of* R^n *with* $0 < H^s(E) < \infty$. *Then for* H^s *almost all* x *in* E *there are arbitrarily small radii* r *such that*

$$H^s(E \cap B(x,r) \cap \{x + y: y/|y| \in S\}) \geq c(b)r^s$$

for any Borel set $S \subset \{y: |y| = 1\}$ *with* $H^{n-1}(S) \geq b$.

Recently Arto Salli has given a different more direct proof for Theorem 1, which also applies to the Minkowski dimension.

Reference:

[M] P. Mattila, Distribution of sets and measures along planes, to appear in J. London Math. Soc.