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APPROXIMATE PEANO DERIVATIVES AND THE BAIRE* ONE PROPERTY

A real valued function f defined on the real line $\mathbb{R}$ is said to have an approximate Peano derivative of order k at x if there are finite numbers $f_{(0)}(x)$, $f_{(1)}(x)$, ..., $f_{(k)}(x)$, and a set E of density one at zero such that

$$\text{(A)} \qquad f(x_o + h) - \sum_{i=0}^{k} \frac{f_{(i)}(x)}{i!} h^i = o(h^k) \quad \text{as } h \to 0,\ h \in E.$$

In this paper we shall insist that $f_{(0)}(x) = f(x)$ so that the notion of approximate continuity at x will correspond to the notion of having an approximate Peano derivative of order 0 at x. If one replaces the expression $o(h^k)$ in (A) by $O(h^k)$, the resulting weaker property is called approximate Peano boundedness of order k at x, thereby paralleling the terminology used by Ash [2] for Peano differentiability and Peano boundedness of order k.

Approximate Peano derivatives are known to share many of the properties of ordinary derivatives and papers investigating these properties include references [4] through [11]. The purpose of this note is to present a proof, using only first principles, that if a function is approximately Peano bounded of order $k + 1$ at each real number, then the k^{th} approximate Peano derivative of the function belongs to the class Baire* one in the notation of