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THE METHOD OF FRACTIONAL OPERATORS APPLIED TO SUMMATION

The purpose of this paper is to show how some interesting results concerning series summation and the psi function are established by means of fractional operators. Our main interest here is the method used to obtain the formula

(1)
$$\psi(\lambda) - \psi(\lambda - \nu) = \frac{\Gamma(\lambda)}{\Gamma(\nu)} \sum_{n=1}^{\infty} \frac{\Gamma(\nu + n)}{n\Gamma(\lambda + n)}$$
, $\operatorname{Re}(\lambda) > \operatorname{Re}(\nu) \ge 0$.

The technique used here to construct relation (1) has been called by the misnomer <u>fractional calculus</u> [1], [2], [3], but we shall refer to it as the method of <u>fractional operators</u>. The great elegance that can be achieved by the proper use of fractional operators should more than justify a more general recognition and use. These operators have the power to simplify the solutions of a complicated functional equations. This paper augments an idea initiated by Ross [4]. We will show that Ross's result is a special case (1) if the parameters v and λ are appropriately specified.