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CONNECTIVITY FUNCTIONS WITH A PERFECT ROAD

Let X and Y be topological spaces. A function $f:X\to Y$ is said to be a connectivity function provided that if A is a connected subset of X, then the graph of f restricted to A is a connected subset of $X\times Y$. A function $f:X\to Y$ is said to be an almost continuous function provided that if O is an open subset of $X\times Y$ containing the graph of f, then there exists a continuous function $g:X\to Y$ such that O contains the graph of g. A real-valued function f defined on an interval is said to have a perfect road at the point x provided that there exists a perfect set P such that x is a bilateral point of accumulation of P and such that f restricted to P is continuous at x.

Let I be the closed unit interval. In order that a function $f:I\to I$ be a connectivity function, it is necessary and sufficient that the graph of the entire function be connected. However, if $f:I^2\to I$ and the entire graph is connected it is not guaranteed that f is a connectivity function. Also, if $f:I\to I$ is an almost continuous function, then f is a connectivity function. But there exist connectivity functions $f:I\to I$ that are not almost continuous, [3], [7], [10]. However, if $f:I^2\to I$ is a connectivity function, then f is an almost continuous function, [11], but the converse is not true. Using these facts a negative answer was given to the