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THE UNIFORM LIMIT OF CONNECTIVITY FUNCTIONS

A. Lindenbaum [4] showed that any real-valued function defined on an interval is the pointwise limit of a sequence of Darboux functions. Phillips [5] obtained the same results with a sequence of functions whose graph is connected and Kellum [3] showed that the same is true for almost continuous functions. In the paper by Kellum an example was given which showed that the uniform limit of a sequence of almost continuous functions $R \to R$ need not be a Darboux function where R is the set of real numbers. Thus the uniform limit of a sequence of connectivity functions need not be a Darboux function. However it must belong to the class of functions characterized by Bruckner, et. al., [2].

Now a natural question arises: "Does there exist a Darboux function which is not the uniform limit of a sequence of connectivity functions?" In this paper we construct a Darboux function $f:I \to I$ which is not the uniform limit of a sequence of connectivity functions where $I = [0,1]$. We also prove the following propositions.

Proposition A. Let X be a metric space. Then the uniform limit f of a sequence $f_m:X \to R$ of peripherally continuous functions is peripherally continuous.