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MONOTONE SECTIONS OF FUNCTIONS OF TWO VARIABLES

We introduce the following notation.

Let $\phi : I \to R$ (where $I = [0,1]$).

If a function ϕ has a property P, we denote this fact by $P(\phi)$.

Let $f : I \times I \to R$. Then for each $x \in I$ we consider $f_x(y) = f(x,y)$ as a function of y and for each $y \in I$ we consider $f^y(x) = f(x,y)$ as a function of x.

We put

$$A_x(f,P) = \{x;P(f_x)\} \quad \text{and} \quad A_y(f,P) = \{y;P(f^y)\}.$$

Let $A_1 \subset I$ and $A_2 \subset I$. We investigate conditions on the sets A_1 and A_2 under which there exists a function f such that $A_1 = A_x(f,P)$ and $A_2 = A_y(f,P)$, where P is a certain fixed property such as "nondecreasing", "increasing", "nondecreasing and continuous", "increasing and continuous", "of bounded variation".

Then we construct a function fulfilling these conditions. At first we suppose that P means "nondecreasing".

Theorem 1. Let $A_1,A_2 \subset I$. Then there exists a function $f(x,y)$ defined on $I \times I$ such that $A_1 = A_x(f,P)$ and $A_2 = A_y(f,P)$ if and only if

1° $I \neq A_1$ and $I \neq A_2$ or

2° $A_1 = A_2 = I$ or

3° $A_1 = I$, $A_2 \neq I$ and $\mathrm{card}(\bar{A}_2 - A_2) \leq \aleph_0$ or
$A_2 = I$, $A_1 \neq I$ and $\mathrm{card}(\bar{A}_1 - A_1) \leq \aleph_0$.

Proof. Sufficiency. If condition 1° or 2° is fulfilled, we define the function $f(x,y)$ in the following way: