

## SELECTIVE DIFFERENTIATION

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Section 0. Introduction.

The author would like to express gratitude to the various editors of the Real Analysis Exchange who suggested this survey article. He hopes that it will be useful in clarifying some of the underlying principles; in particular, the relationships between selective, biselective and path system derivatives.

A selection can be thought of as either an interval function or a point function. As an interval function, a selection consists of picking one point from the interior of each nondegenerate subinterval  $[a,b]$  of  $\mathbb{R}$ . (Throughout this paper,  $[a,b]$  denotes the interval with endpoints  $a$  and  $b$  even if  $a > b$ .) However, it is sometimes useful to consider a selection as a point function. Then a selection  $s$  is a function whose domain is the upper half plane  $U = \{ (x,y) : x < y \}$  and which satisfies the relation  $x < s(x,y) < y$ .

Section 1. History.

Motivation for the basic concepts of selective differentiation came primarily from papers [G1., G.-N., N., S.1, S.2, B] due to Gleyzal, Goffman, Neugebauer, Snyder, and Bruckner.

Gleyzal [G1] said that an interval function  $\phi$ , defined on the collection of all nondegenerate compact subintervals of  $\mathbb{R}$ , is convergent to a point function  $f$  if and only if for each  $x$   $\lim_{I \rightarrow x} \phi(I) = f(x)$ , where  $I \rightarrow x$