A New Approach to Integration

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1. <u>Introduction</u>. Recently ([7], [8], [10], [13]) a
new theory of integration has been developed. It is
more general than the Lebesgue integral [3] but it
retains the monotone and dominated convergence properties
of that integral. Unlike the Riemann and Lebesgue
integrals, it always retrieves a function from its
everywhere finite derivative:

$$\int_a^b F'(x)dx = F(b) - F(a).$$

In fact, slightly more is true (see Theorem 2.1 below).

This "new" integral is really not new - it is equivalent
to the Perron and the restricted Denjoy integrals introduced
early this century ([18] and [7]), i.e., a function is
integrable in the new sense iff it is Perron and restricted
Denjoy integrable, in which case the values of the integrals
are all the same. What is new and remarkable is the
definition of this integral which is very elementary,
namely, a slight modification of Riemann's. In accordance
with present convention we shall refer to this integral as
"generalized Riemann integral". The status of Lebesgue
integrability is easily described: f is Lebesgue integrable