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HIGH ORDER SMOOTHNESS

In the talk given at the Louisville Symposium and in this summary of that presentation, all functions are assumed to be Lebesgue measurable real valued functions defined on the real line $\mathbb{R}$. The classical notion of smoothness is that f is smooth at x provided

$$f(x+t)+f(x-t)-2f(x)=o(t) \text{ as } t \to 0,$$

and f is called a smooth function if it is smooth at each point $x \in \mathbb{R}$.

The following theorem is a summary of several known properties of smooth functions. It is essentially due to C. J. Neugebauer [4], although several of its parts have been proved by earlier authors using more restrictive hypotheses.

Theorem A. *Let $f\colon \mathbb{R} \to \mathbb{R}$ be smooth. Then*

a) *f is in class Baire* one (every perfect set P contains a portion Q such that the restriction of f to Q is continuous.)*

b) *if $E = \{x : f'(x)$ exists and is finite$\}$, then E is c-dense.*

c) *if f has the Darboux property on $\mathbb{R}$, then*

   i) *f' has the Darboux property on E*

   ii) *if $f'(x) \geq 0$ for each x in E, then f is increasing and continuous on $\mathbb{R}$.*

The notion of smoothness naturally extends to the $L_p (1 \leq p < \infty)$ setting by saying that f is L_p smooth at x provided