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TWO MORE CHARACTERIZATIONS OF BESOV-BERGMAN-LIPSCHITZ SPACES

Dedicated to Francisco Vieira de Sales on his 100th Birthday

In the early 1960's the following spaces were introduced, now known as Besov spaces. For $0 < \alpha < 1$, $1 < r$, $s < \infty$, let

$$\Lambda(\alpha, r, s) = \{f: [-\pi, \pi] \rightarrow \mathbb{R} ; \|f\|_{\Lambda(\alpha, r, s)} = \|f\|_r + \left(\int_{-\pi}^{\pi} \frac{(\|f(x+t) - f(x)\|_r)^s}{|t|^{1+\alpha s}} dt \right)^{1/s} < \infty \}.$$

where $\| \cdot \|_r$ is the Lebesgue space L^r -norm. For these spaces the reader is referred to [1], [7], [8] and [9].

Notice that $\Lambda(\alpha, \infty, \infty)$ is the usual Lipschitz spaces.

The following spaces of analytic functions on the disk have been studied in depth by several people, for example by E. Stein, M. Taibleson, A. L. Shields, and others.

$$J^p = \{g: D \rightarrow \mathbb{C}, \text{ Analytic, } \|g\|_{J^p} = |g(0)| + \frac{1}{\pi} \int_0^1 \int_{-\pi}^{\pi} |g'(re^{i\theta})| (1-r)^{\frac{1}{p}-1} d\theta dr < \infty\}$$

for $1 \leq p < \infty$. The dash means derivative.

In [9], M. Taibelson has shown that $\Lambda(1 - \frac{1}{p}, 1, 1)$ is equivalent as Banach spaces to J^p for $1 < p < \infty$.

In these notes we propose to give two very simple characterizations of the spaces $\Lambda(1 - \frac{1}{p}, 1, 1)$ and J^p for $1 < p < \infty$, in terms of non-increasing functions. For $p = 1$ we also get a result.