CONCERNING EXTENDABLE CONNECTIVITY FUNCTIONS

By Jerry Gibson

In the classic paper [16], J. Stallings asked the following question: "If $I = [0,1]$ is embedded in I^2 as $I \times \{0\}$, can a connectivity function $f: I \rightarrow I$ be extended to a connectivity function $g: I^2 \rightarrow I$?" Negative answers were given to this question by Cornette [4] and Roberts [14]. Each constructed a connectivity function $I \rightarrow I$ that is not an almost continuous function.

Definition 1. $f: X \rightarrow Y$ is a connectivity function if and only if the graph of f restricted to C is connected in $X \times Y$ whenever C is connected in X.

Definition 2. $f: X \rightarrow Y$ is an almost continuous function if and only if each open subset of $X \times Y$ containing the graph of f contains the graph of a continuous function with the same domain.

In this paper all propositions will be restricted to I, I^2, or $I \times \{p\}$ where $p \in I$ even though they may have been proved (or maybe proved) more generally.

Proposition 1. If $f: I \rightarrow I$ is an almost continuous function, then f is a connectivity function, [16].

Proposition 2. If $f: I^2 \rightarrow I$ is a connectivity function, then f is an almost continuous function, [16].

Proposition 3. $f: I \rightarrow I$ is a connectivity function if and only if the entire graph of f is connected.