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Products of Approximate Derivatives

In (2) and (3) a characterization is given for the class of functions defined on $[0,1]$ whose product with every derivative is a derivative. In the present note characterizations of the multiplier class for approximate derivatives of continuous functions (ADC) and the multiplier class for the approximate derivatives (AD) are given and their proofs are sketched.

For a function $F(x)$, let $W(F,I)$ denote its total variation on the interval I and let $W(t) = W(F, [0,t])$.

Theorem 1. A function $F(x)$ belongs to ADC if and only if $W(t)$ satisfies a Lipschitz condition at each point x in $[0,1]$.

(Stated in the form of a Stieltjes integral, this condition requires that $\left|\int_{x_0}^{x} dW(t)\right| \leq M_{x_0} \cdot |x - x_0|$.)
It is interesting to compare this with the notion of distant bounded variation, $\left|\int_{x_0}^{x} (t - x_0) dW(t)\right| \leq M_{x_0} \cdot |x - x_0|$, which characterizes the multiplier class for ordinary