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ON SOME RINGS OF SWIATKOWSKI FUNCTIONS

In 1977, T. Mank and T. Swiatkowski in paper [1] defined a new class of functions. According to the terminology adopted in [2] elements of this class we call Swiatkowski functions.

Defintion. We say that $f:R \to R$ is a Swiatkowski function if for every two points $x,y \in R$ such that $f(x) \neq f(y)$ there exists a point z of continuity of f such that $z \in (x,y)$ and $f(z) \in (f(x),f(y))$.

We assume the notation (a,b) in either case $a < b$ or $b < a$.

Let C_f (D_f) denote the set of all continuity (discontinuity) points of f.

It is known that there exist Swiatkowski functions f and g such that $f + g$ is not a Swiatkowski function. So the question whether it is possible to form a ring of Swiatkowski functions, containing all continuous functions and a fixed Swiatkowski function f, seems to be interesting.

For a Swiatkowski function $f:R \to R$ let $RS(f)$ denote the class of all complete rings K of Swiatkowski functions such that $f \in K$ and $C \subset K$, where C denotes the class of all continuous functions. (A ring K of real functions is complete if for every $g \in K$, $|g|$ also belongs to K.)

Now the above question can be formulated in the following way: Under what hypothesis on f is $RS(f) \neq \emptyset$?