INROADS Real Analysis Exchange Vol. 9 (1983-84)

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On Generalized Cluster Sets

The paper consists of three parts. In the first we consider σ-ideals of subsets of the plane adjoint with some σ-ideal $\mathcal{J}$ of subsets of the real line. The second part contains some theorems concerning σ-algebras of the form $(\mathcal{B} \Delta \mathcal{J})^2$, where $\mathcal{B}$ is a σ-algebra of Borel sets. In the third part the facts from the two earlier parts are used to study generalized limit numbers of real function defined in the upper half-plane.

1. Let H denote the open upper half-plane above the real line R, S - a σ-algebra of subsets of R and S^2 - the smallest σ-algebra generated by sets $A \times B$, where $A \in S$ and $B \in S$. $L(x,\theta)$ is the halfline beginning at $x \in R$ in the direction θ, $L(x,\theta,r)$ - the segment beginning at $x \in R$ in the direction θ having length r. For $x \in R$ let h_x be the real function defined in H such that $h_x(p) = r$ for $p \in H$, where r is the distance of p from x.

For any σ-ideal $\mathcal{J} \subset S$ and direction $\theta \in (0,\pi)$ we shall define the σ-ideal $\mathcal{J}^2(\theta)$ adjoint with $\mathcal{J}$ in the direction θ:

$$\mathcal{J}^2(\theta) = \{M \in S^2 : \text{there is a set } U \in \mathcal{J} \text{ such that}$$

$$h_x(L(x,\theta) \cap M) \in \mathcal{J} \text{ for each } x \in R - U\}$$